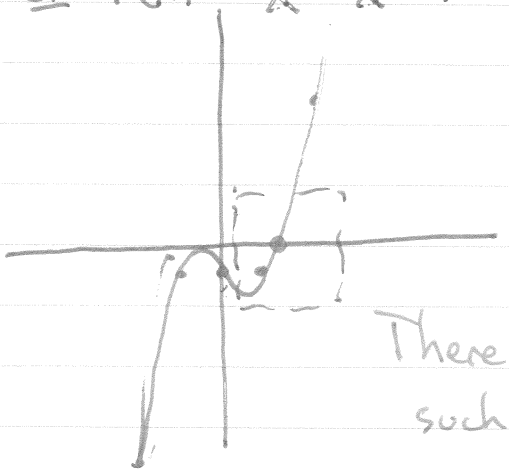
Newton's Method:

Q: $f(x) = x^3 - x - 1$

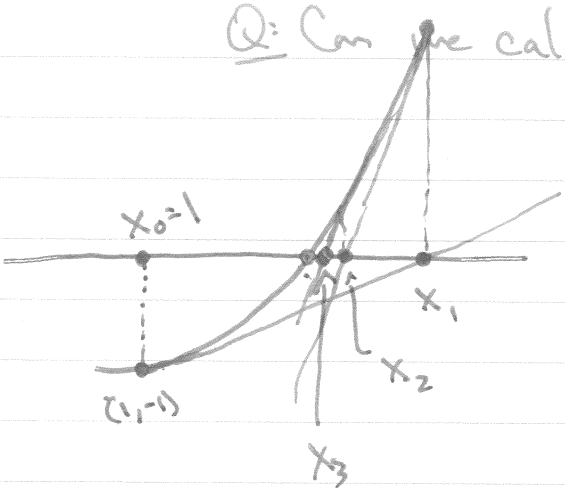


x	$f(x)$
-2	-7
-1	-1
0	-1
1	-1
2	5

There is a point x between 1 & 2 such that $f(x)=0$

Q: Can we calculate this root?

Newton's Method:



- Guess $x_0 = 1$
- Draw tangent line to $y=f(x)$ at $x=x_0$, and see where it intersects x-axis. x_1

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 1$

$x_1 = 1 - \frac{1^3-1-1}{3\cdot 1^2-1}$

$= 1.5$

- Draw tangent at $x=x_1$, intersects x-axis at x_2.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

- x_2

  x_3

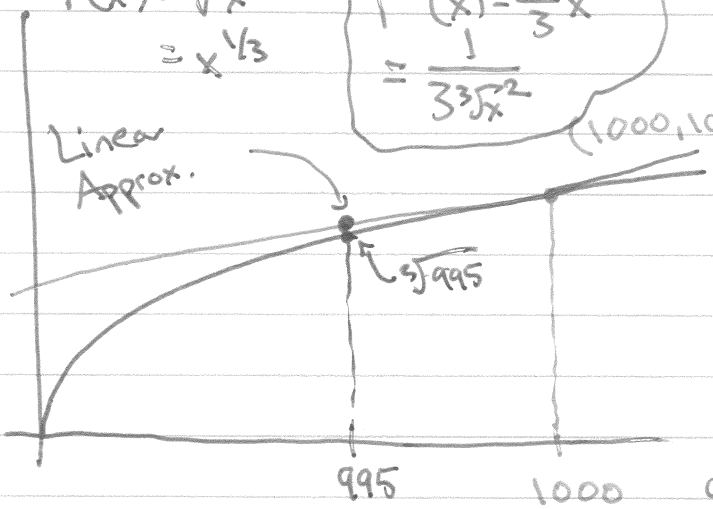
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

⋮

Approximate $\sqrt[3]{995}$

$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$



$x = a$ should satisfy:
- Know $f(a)$, $f'(a)$
- Needs to be close to 995

$a = 1000$

Equation of Tangent Line

$$y = f(a) + f'(a)(x-a)$$

$a = 1000$

$$y = f(1000) + f'(1000)(x-1000)$$
$$= 10 + \frac{1}{3\cdot 10^2}(x-1000)$$
$$= 10 + \frac{1}{300}(x-1000)$$

$\frac{1}{3\sqrt[3]{1000}^2}$

Plug $x = 995$. $\rightsquigarrow 10 + \frac{995-1000}{300}$

$$= \boxed{10 - \frac{1}{60}} = \boxed{9.9833} = \boxed{\frac{59}{60}}$$

This is an overestimate because $\sqrt[3]{x}$ has 2nd derivative negative.

Quiz 1: Mean - 12.1
Std Dev - 3.2

Midterm: Sign-ups on Wiki

$f(x) = \frac{1}{\sqrt{x^2+2}}$ $\quad f'(x) = ?$

$\frac{-1}{2(x^2+2)^{3/2}}$ $\quad \boxed{\frac{-x}{(x^2+2)^{3/2}}}$

$\frac{1}{\sqrt{x^2+2}} = \frac{1}{(x^2+2)^{1/2}} = (x^2+2)^{-1/2}$

Chain Rule: $g(x) = x^2+2$
$h(x) = x^{-1/2}$ $\quad = h(g(x))$

$\Rightarrow f'(x) = \underbrace{h'(g(x))}\ \underbrace{g'(x)}$

$-\frac{1}{2}(x^2+2)^{-3/2}\ 2x = \frac{-x}{(x^2+2)^{3/2}}$

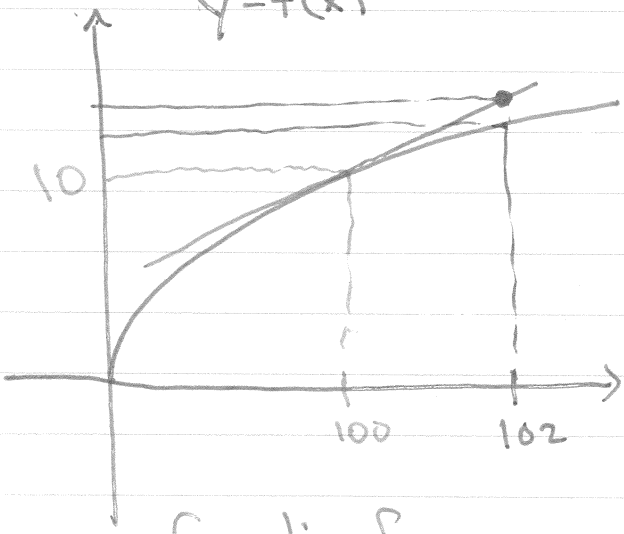
Last time: Linear Approximation.

Ex. $\sqrt{102} = ?$

$y = \sqrt{x}$

We know $\sqrt{100} = 10$

Draw the tangent line at $x=100$



Because 100 & 102 are close, the value of y at $x=102$ for the tangent line is a good approx. for $\sqrt{102}$

Equation for Tangent Line to $y=f(x)$ at $x=a$:

$y = f(a) + f'(a)(x-a)$ (slope)

$(a, f(a))$ is on the line.